Equations used to model yeast growth in bioreactor during exponential growth of yeast.

As long as there is plenty of glucose and oxygen, the growth of yeast is proportional to the amount of yeast present.

$$\frac{dY}{dt} = K_y Y \qquad (1)$$

where Y in the concentration of yeast, t is time and K_y is the specific growth rate of the yeast. Integration of equation 1 yields

$$Y = Y_0 \exp[K_y (t - t_0)] \qquad (2)$$

where Y_0 is the concentration of yeast at $t = t_0$. (Remember t_0 must be longer than the incubation period.)

The change of glucose with time is proportional to the amount of yeast present.

$$\frac{dG}{dt} = -K_G Y = -K_G Y_0 \exp[K_y (t - t_o)] \qquad (3)$$

Where K_G is the specific glucose uptake rate. Integration of equation 3 yields

$$G = G_1 - \frac{K_G Y_0}{K_y} \left\{ \exp\left[K_y (t - t_0)\right] - \exp\left[K_y (t_1 - t_0)\right] \right\} \qquad (4)$$

Where G_1 is the concentration of glucose at time = t_1. t_1 can be = to t_0 or any time when you know a corresponding glucose concentration, G_1.

Note that equations 1 through 4 are applicable for all times greater than the incubation time.

An oxygen balance over this time **when the mixture is injected with air** gives

$$\begin{aligned} \frac{dO_2}{dt} &= \left(K_m a + \frac{K_s}{h}\right)(O_2^{sat} - O_2) - K_O Y \\ &= \left(K_m a + \frac{K_s}{h}\right)(O_2^{sat} - O_2) - K_O Y_0 \exp[K_y (t - t_0)] \end{aligned} \qquad (5)$$

Where K_m is the mass transfer coefficient for the air in the bubbles, a is the interfacial area per unit volume, Ks is the mass transfer coefficient to the top surface of the liquid, h is the height of the liquid in the reactor, O_2^{sat} is the concentration of O_2 that would be in the solution if it were in equilibrium with the O_2 in the air and K_O is the specific oxygen uptake rate.

Equation 5 is a linear first order equation. The solution of this equation can be found using an integration factor. This solution of this equation is.

$$\theta = \theta_2 \exp\left[-\left(K_m a + \frac{K_s}{h}\right)(t - t_2)\right] - \frac{K_O Y_0}{K_y + K_m a + \frac{K_s}{h}}\left\{\exp\left[K_y(t - t_0)\right] - \exp\left[K_y\left(t_2 - t_0\right) - \left(K_m a + \frac{K_s}{h}\right)(t - t_2)\right]\right\} \quad (6)$$

Where $\theta = O_2 - O_2^{sat}$ and $\theta_2 = \theta$ at time = t_2. The units for O_2 are mol/m^3, for K_m and K_s are m/s, and for a and $1/h$ and m^{-1}. O_2^{sat} can be found from Henry's Law for solubility of oxygen in water. It is a function of temperature and atmospheric pressure. Remember equation 6 is valid when the air is bubbling through the solution. T_2 can be chosen for your convenience, but must be different for each section of the curve.

When the air is turned off and N_2 blankets the surface of the mixture, there is some mass transfer of O_2 from the liquid to the N_2 and an O_2 balance assuming there is no O_2 in the N_2 blanket ($O_2^{sat} = 0$) is

$$\frac{dO_2}{dt} = -\frac{K_s}{h} O_2 - K_O Y_0 \exp[K_y (t - t_0)] \quad (7)$$

Integration of this equation leads to:

$$O_2 = O_2(t_3)\exp\left[-\frac{K_s}{h}(t - t_3)\right] - \frac{K_O Y_0}{K_Y + \frac{K_s}{h}}\left\{\exp\left[K_y\left(t - t_0\right)\right] - \exp\left[K_Y(t_3 - t_0) - \frac{K_s}{h}(t - t_3)\right]\right\} \quad (8)$$

Again, equation 8 is only valid when the O_2 is turned off. The time t_3 can be chosen any time when the O_2 is off, but it is probably best to select it as the time when the O_2 is first turned off. It is different for each time the O_2 is turned off.

In summary:

1. Use equation 2 to get K_y. This equation is valid over the entire time of the experiment when the yeast is in its exponential growth rate.
2. Once K_y has been determined, use equation 4 to find K_G. This equation is valid for the entire exponential growth period.
3. Use equation 8 to find K_O and K_s/h This equation is only valid when the O_2 is off. Use all the data from each of these times to find K_O and K_s/h. Each time will have a different t_3.
4. Once K_y, K_O, and K_s/h have been found, use equation 6 to find $K_m a$. You can probably do this best by using the solver function in Excel. Use all the time

periods when the O_2 is on to find the best $K_m a$. Each time period will have a different t_2.

For extra credit, you can run an experiment when you do not put an N_2 blanket over the reactor when the air is turned off and see if the parameters found in the above experiments reproduce your results. Note, equation 8 will need to be modified, but no new parameters will need to be found.