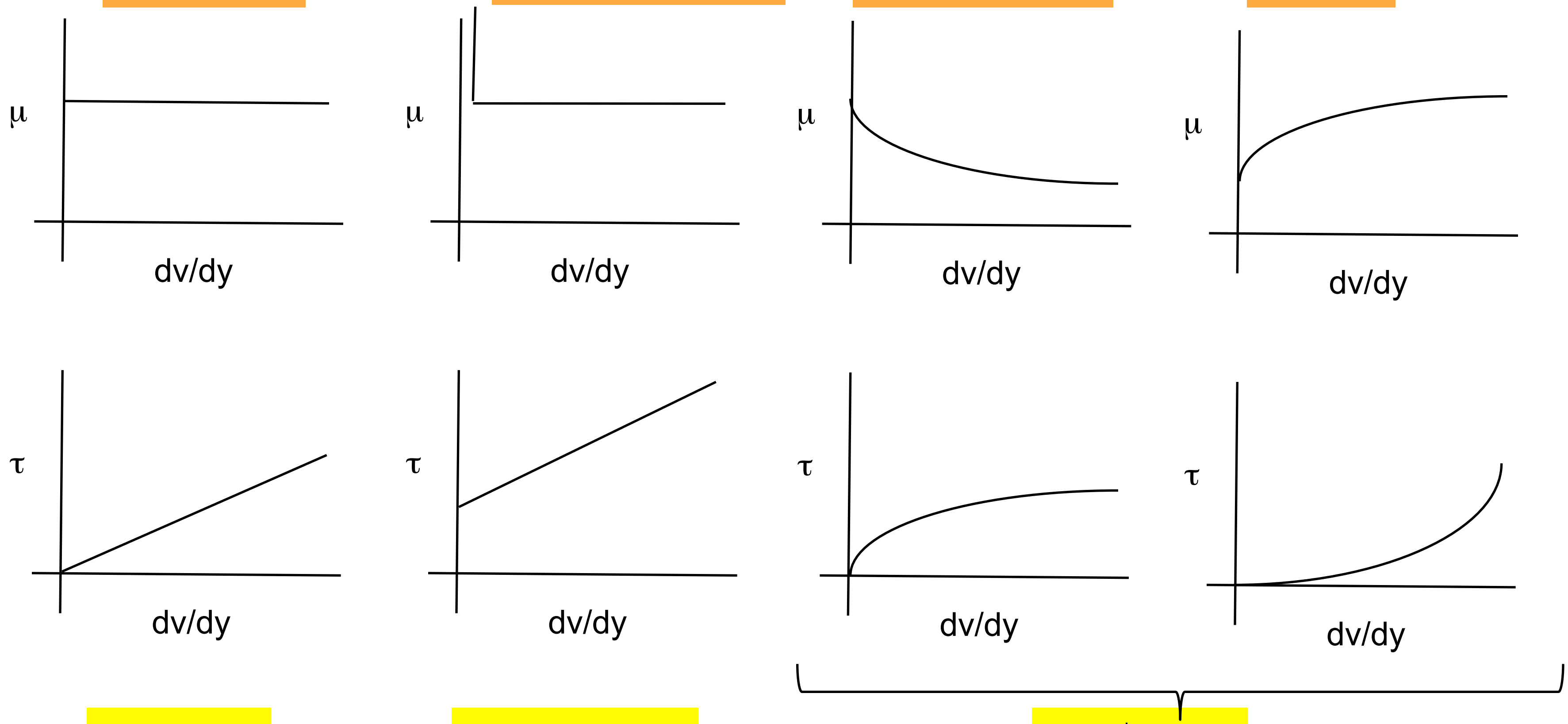
Chemical Engineering 374

Fluid Mechanics
Fall 2009

NonNewtonian Fluids

Non-Newtonian Fluids

Newtonian | **Bingham Plastic** | **Pseudoplastic** | **Dilitant**



$\tau = \mu * dv/dy$

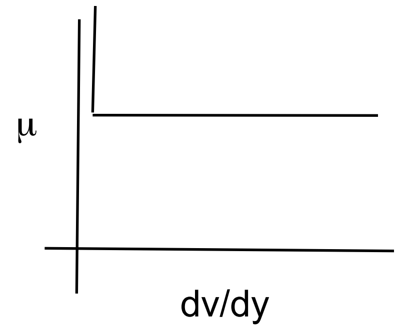
$\tau = \mu * dv/dy + \tau_y$

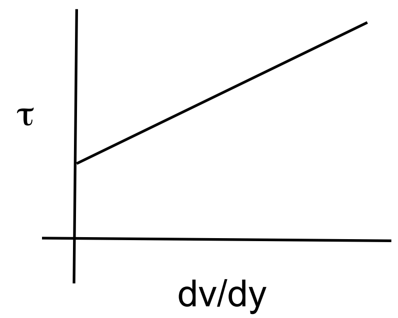
$\tau = \kappa * |dv/dy|^n$

Bingham Plastic

- 3D elastic structures.
- Resists small shear, but structure “breaks apart” with large shear.
- Then τ is ~ linear with du/dx
- Coal slurries, grain slurries, sewage sludge.
- Larger particles → weak solid structure → breaks

Bingham Plastic

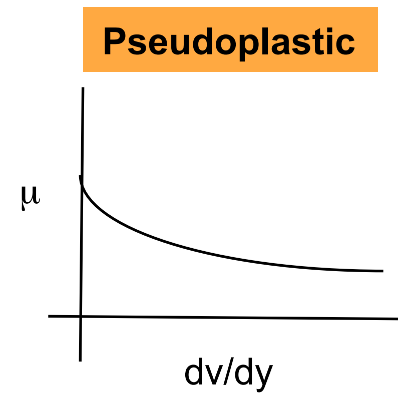


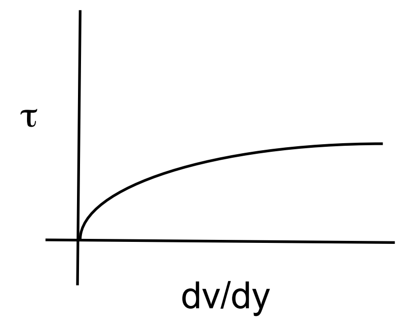


$\tau = \mu * dv/dy + \tau_y$

Pseudoplastic

- Most common
- Dissolved or dispersed particles, like dissolved long chain molecules.
- Have a random orientation in the fluid at rest, but line up when the fluid is sheared.
  - τ decreases with strain rate
  - μ drops as molecules align
- Polymer melts, paper pulp suspensions, pigment suspensions, hair gel.
- “Shear-Thinning”



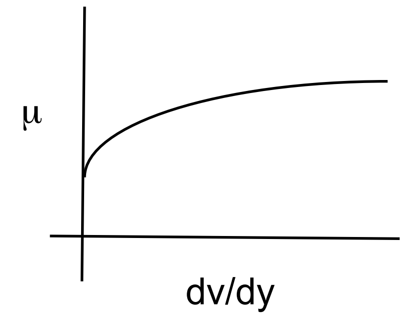


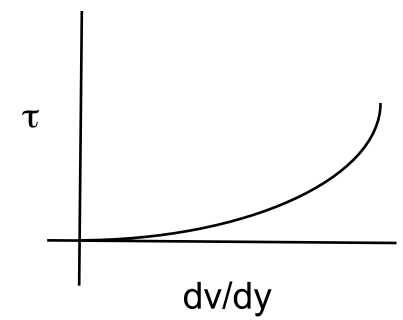
$\tau = K * (-dv/dy)^n$

Dilitant

- Rare
- Slurries of solid particles with barely enough liquid to keep apart.
- At low strain rates, the fluid can lubricate solids; at high strain rates, this lubrication breaks down.
- μ increases with strain → τ increases.
- Corn starch.
- “Shear thickening”.

Dilitant



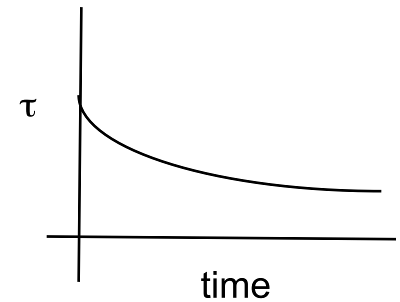


$\tau = K * (-dv/dy)^n$

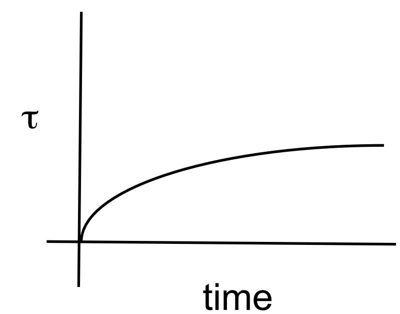
Non-Newtonian

Time dependence

- Thixotropic
  - Slurries/solutions of polymers
  - Many known fluids
  - Most are pseudoplastic
  - Alignable particles/molecules with weak bonds (H-bonding)



- Rheopectic
  - Rare
  - Fewer known examples
  - Usually fluids only show this behavior under mild shearing



- Changes occur within the first 60 sec. for most processes.
- Hard to describe

Outline

Last Class: Types and Properties of non-Newtonian Fluids with Examples

- Pipe flows for non-Newtonian fluids
- Velocity profile / flow rate
- Pressure drop
  - Friction factor
  - Pump power
- Rheological Parameters

Power Law Fluids

Power Law Fluids

- Governing equations are "correct" in terms of τ
  - Expression for τ is the model.
  - Called a "constitutive relation"
    - Also have these for mass and heat fluxes in heat and mass transfer.
  - Newtonian flow

$$\tau = -\mu \frac{dv}{dy}$$

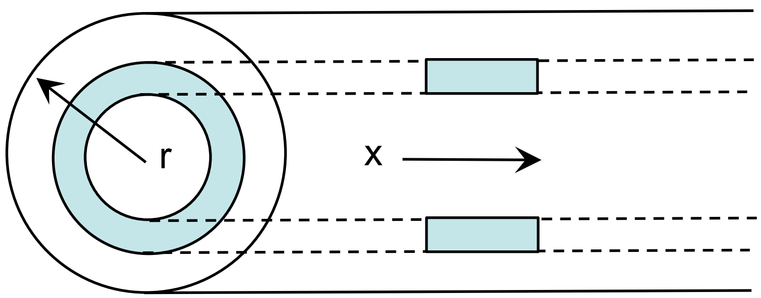
  - For dilitant and pseudoplastic fluids (most common)—***Power Law***

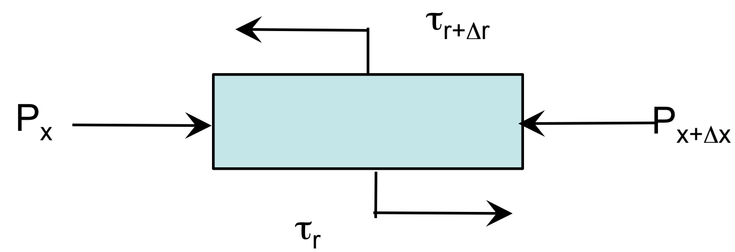
$$\tau = K\left(-\frac{dv}{dy}\right)^n$$

n>1 → Dilitant
n<1 → Psuedoplastic
n=1, K=μ → Newtonian

    - K, n are empirical constants
  - Many other forms
    - Simpler ones have 3 parameters and give a better fit, but are more complex than power law form.
      - See Handout of Book Chapter on Blackboard.

Laminar Pipe Flow





Force Balance: Pressure, stress

$$(P_x - P_{x+\Delta x})(2\pi r \Delta r) + (2\pi \Delta x r)\tau_r - (2\pi \Delta x)(r + \Delta r)\tau_{r+\Delta r} = 0$$

Divide $2\pi \Delta r \Delta x$

$$r\frac{P_x - P_{x+\Delta x}}{\Delta x} + \frac{r\tau_r - (r + \Delta r)\tau_{r+\Delta r}}{\Delta r} = 0$$

Limit Δx, $\Delta r \rightarrow 0$

$$-\frac{dP}{dx} = \frac{1}{r}\frac{d(r\tau)}{dr} = C$$

Separate variables and integrate with τ=0 at r=0

$$\tau = -\frac{r}{2}\frac{dP}{dx}$$

Non-Newtonian Pipe Flow

- Most non-Newtonian flows are laminar.
- Key results:
  - Force balance:
  - Power law constitutive relation
  - Integrate with B.C. v=0 at r=R

$$\tau = -\frac{r}{2}\frac{dP}{dx}$$

$$\tau = K\left(-\frac{dv}{dr}\right)^n$$

$$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

  - $Q = Av_{avg}$

$$Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n} \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

  - Kinetic Energy Correction Factor:

$$\alpha = \frac{3(3n+1)^2)}{(5n+3)(2n+1)}$$

  - Momentum Flux Correction Factor:

$$\beta = \frac{3n+1}{2n+1}$$

Pressure Drop

- Again $\tau = -\frac{r}{2}\frac{dP}{dx} \longrightarrow \tau_w = -\frac{R}{2}\frac{dP}{dx}$
- Recall definition of friction factor: $f = \frac{8\tau_w}{\rho V_{avg}^2}$
- Insert τ_w, with dP/dx = ΔP/L, and rearrange: $\frac{\Delta P}{\rho} = -F = -\frac{fLV_{avg}^2}{2D}$
  - This gives the same pressure drop equation as for laminar flow
    - (What about turbulent flow)
  - Pump power is just $\dot{W} = -Q\Delta P = -\dot{m}\frac{\Delta P}{\rho} = \dot{m}F$
- f for non-Newtonian fluids:
  - Laminar pipe flow: $V_{avg} = \frac{nD}{4(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$
  - Solve for dP/dx = ΔP/L and insert into $\frac{\Delta P}{\rho} = -F = -\frac{fLV_{avg}^2}{2D}$
  - Solve result for f: $f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n$

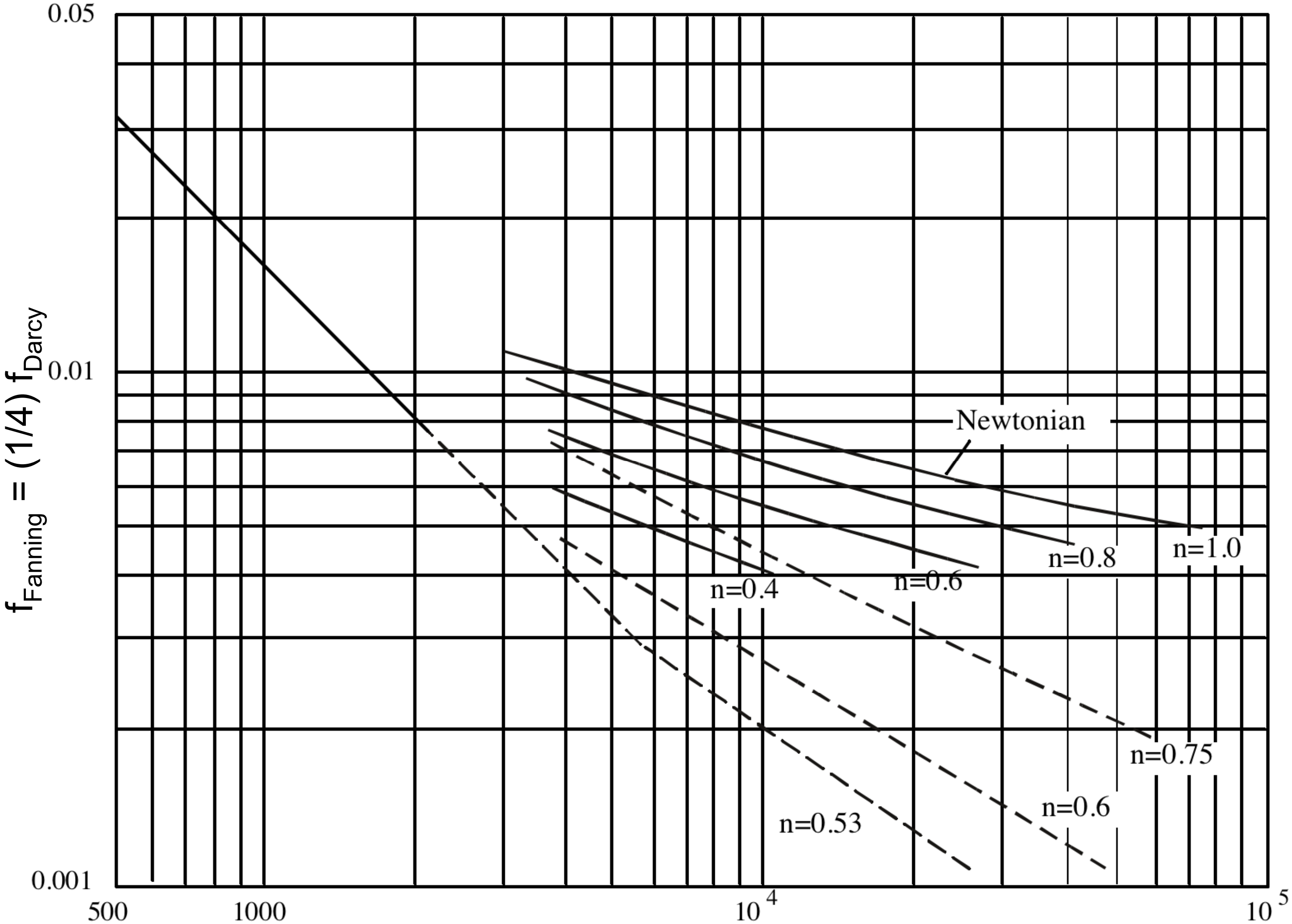
Turbulent Flow

- Define the friction factor as before: $$f = \frac{8\tau_w}{\rho V_{avg}^2} = \frac{\Delta P D/L}{\frac{1}{2}\rho V_{avg}^2}$$
  - (Laminar or Turbulent)
- For turbulent flow we had f = f(Re, ε/D) from dimensional analysis.
- Question: Will this work for non-Newtonian Flow?
- Question: What is the Reynolds number?
  - No clear definition of Re since there μ is not constant (depends on the strain rate dv/dr, which depends on V_{avg}
- Use the same definition as the laminar friction factor: Re=64/f

$$f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n \longrightarrow Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n$$

  - (Poiseuille's Law)
  - (Definition based on laminar Newtonian, but used for all regimes)
- Plot friction factor versus Re as for Newtonian flows

Non-Newtonian Friction Factor (Power Law)



$$Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n$$

Rheological Parameters (power law)

- Analyse a non-Newtonian fluid from data: **Compute n, K given data**
- **Measure** D, Q, dP/dx → τ_w, v_{avg}.
- **Now:** $\tau = K\left(-\frac{dv}{dr}\right)^n \longrightarrow \ln(\tau_w) = \ln(K) + n\ln(-dv/dr)_w$
- With $\left(-\frac{dv}{dr}\right)_w = \frac{2(3n+1)V_{avg}}{nD}$ computed from our v(r) solution.
- A plot of ln(t_w) versus ln(-dv/dr)$_w$ is linear with slope n, but to get (-dv/dr)$_w$ we need n.
- Instead: **Define** $\xi = \frac{8V_{avg}}{D}$ (happens to be (-dv/dr)$_w$ for Newtonian flow)
  - We know this up front.
- Now $\left(-\frac{dv}{dr}\right)_w = \xi\frac{(3+1/n)}{4}$
- Now insert this into $\ln(\tau_w) = \ln(K) + n\ln(-dv/dr)_w$ to get

$$\ln(\tau_w) = n\ln(\xi) + [n\ln((3+1/n)/4) + \ln(K)]$$

  - Plot ln(τ_w) versus ln(ξ), which has slope n.
  - The term in brackets is the intercept. Once n is found, compute K from the intercept (or analytically).
  - Units on K are kg*s^{n-2}/m from definition of power-law stress.
- **Once n is found as slope of ln(τ_w) versus ln(ξ), K is computed analytically as**

$$K = \frac{\tau_w}{\left(\frac{\xi}{4}(3+1/n)\right)^n}$$

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Example

- Given:
  - Diameter
  - Pressure Drop
  - Flow Rate
- Compute:
  - K, n
  - Re
  - Power through a given pipe