Chemical Engineering 374

Fluid Mechanics

NonNewtonian Fluids

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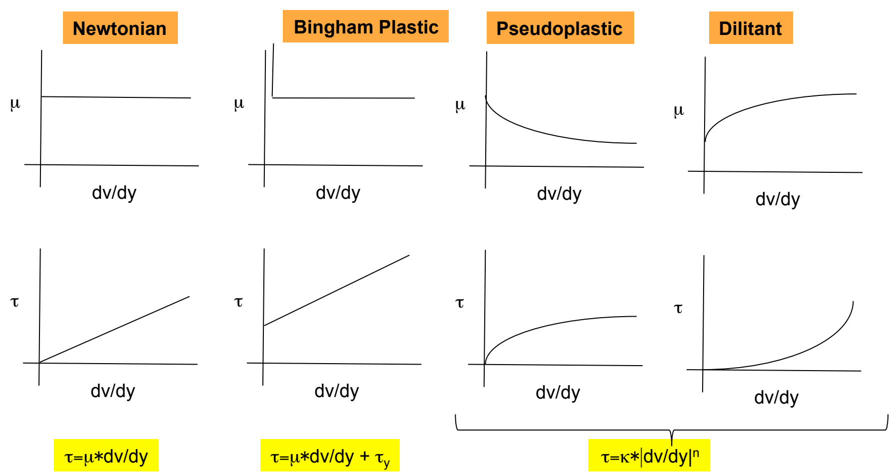
Outline

- Types and properties of non-Newtonian Fluids
- Pipe flows for non-Newtonian fluids
- Velocity profile / flow rate
- Pressure drop
  - Friction factor
  - Pump power
- Rheological Parameters

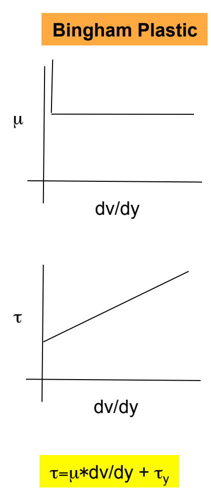
Power Law Fluids (applies to: Velocity profile / flow rate; Pressure drop; Friction factor; Pump power; Rheological Parameters)

Non-Newtonian Fluids



Bingham Plastic

- 3D elastic structures. Weak solid structures that must be broken
- Resists small shear, but structure "breaks apart" with large shear.
- Then τ is ~ linear with du/dx
- Some slurries (coal, grain slurries), sewage sludge.
  - Toothpaste (no drip)
- Larger particles → weak solid structure → breaks



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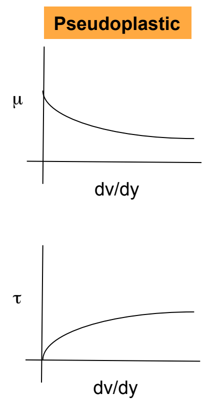
Pseudoplastic

- Most common
- Dissolved or dispersed particles, like dissolved long chain molecules.
- Have a random orientation in the fluid at rest, but line up when the fluid is sheared.
  - τ decreases with strain rate
  - μ drops as molecules align
- Polymer melts, paper pulp suspensions, pigment suspensions, hair gel, blood, muds, most slurries
- "Shear-Thinning"
  - motor oil



$\tau = K*(-dv/dy)^n$

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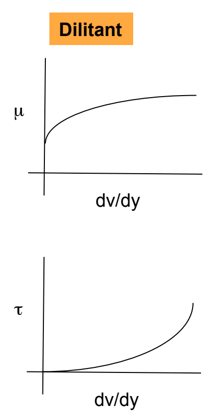
Dilitant

- Rare
- Slurries of solid particles with barely enough liquid to keep apart. (corn starch, water squeezed out at high shear)
- At low strain rates, the fluid can lubricate solids; at high strain rates, this lubrication breaks down.
- μ increases with strain → τ increases.
- "Shear thickening".

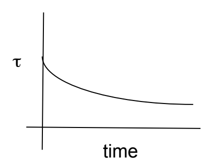


$\tau = K*(-dv/dy)^n$

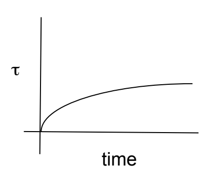
Non-Newtonian

Time dependence

- Thixotropic
  - Slurries/solutions of polymers
  - Many known fluids
  - Most are pseudoplastic
  - Alignable particles/molecules with weak bonds (H-bonding)
  - Paint



- Rheopectic
  - Rare
  - Fewer known examples
  - Usually fluids only show this behavior under mild shearing



- Changes occur within the first 60 sec. for most processes.
- Hard to describe
- Viscoelastic

Power Law Fluids

- Governing equations are "correct" in terms of τ
  - Expression for τ is the model.
  - Called a "constitutive relation"
    - Also have these for mass and heat fluxes in heat and mass transfer.
  - Newtonian flow

$$\tau = -\mu \frac{dv}{dy}$$

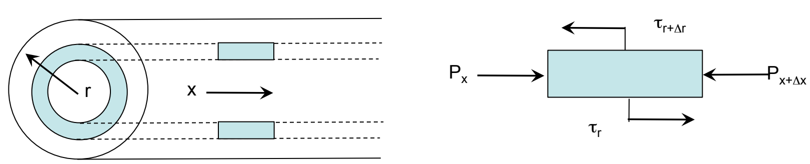
  - For dilitant and pseudoplastic fluids (most common)—***Power Law***

$$\tau = K\left(-\frac{dv}{dy}\right)^n$$

n>1 → Dilitant
n<1 → Psuedoplastic
n=1, K=μ → Newtonian

    - K, n are empirical constants
  - Many other forms
    - Simpler ones have 3 parameters and give a better fit, but are more complex than power law form.
      - See Handout of Book Chapter on Webpage.

Laminar Pipe Flow



Force Balance: Pressure, stress

$$(P_x - P_{x+\Delta x})(2\pi r\Delta r) + (2\pi\Delta x r)\tau_r - (2\pi\Delta x)(r+\Delta r)\tau_{r+\Delta r} = 0$$

Divide $2\pi\Delta r\Delta x$

$$r\frac{P_x - P_{x+\Delta x}}{\Delta x} + \frac{r\tau_r - (r+\Delta r)\tau_{r+\Delta r}}{\Delta r} = 0$$

Limit Δx, Δr → 0

$$-\frac{dP}{dx} = \frac{1}{r}\frac{d(r\tau)}{dr} = C$$

Separate variables and integrate with τ=0 at r=0 $\quad \tau = -\frac{r}{2}\frac{dP}{dx}$

Non-Newtonian Pipe Flow

- Most non-Newtonian flows are laminar.
- Key results: (remember, Q is just volumetric flow rate-Vdot)
  - Force balance: $\tau = -\frac{r}{2}\frac{dP}{dx}$
  - Power law constitutive relation: $\tau = K\left(-\frac{dv}{dr}\right)^n$
  - Integrate with B.C. v=0 at r=R: $v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$
  - Q = Av_{avg}
    - Q is volumetric flow rate

$$Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n} \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

  - Kinetic Energy Correction Factor: $\alpha = \frac{3(3n+1)^2)}{(5n+3)(2n+1)}$
  - Momentum Flux Correction Factor: $\beta = \frac{3n+1}{2n+1}$

Pressure Drop—Laminar Flow

Define $f = \frac{4\tau_w}{\frac{1}{2}\rho v_{avg}^2}$ **1**

Force Balance $\frac{dP}{dx} = -\frac{2\tau_w}{R}$ **2**

$$V_{avg} = -\frac{1}{\pi R^2}\int_A v(r)dA$$

Newtonian		***Non-Newtonian***
$v = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$	**3**	$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$
$V_{avg} = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$	**4**	$V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$
$\tau_w = \frac{4\mu V_{avg}}{R}$ ⟵	**Solve 4 for dP/dx and insert into 2**	⟶ τ_w=(some complex expression)
$f = \frac{64}{Re}$ ⟵	**Insert into 1 for τ_w**	⟶ $f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n$

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Turbulent Flow

- Define the friction factor as before: $f = \frac{8\tau_w}{\rho V_{avg}^2} = \frac{\Delta PD/L}{\frac{1}{2}\rho V_{avg}^2}$
  - (Laminar or Turbulent)
- For turbulent flow we had f = f(Re, ε/D) from dimensional analysis.
- Question: Will this work for non-Newtonian Flow?
- Question: What is the Reynolds number?
  - No clear definition of Re since μ is not constant (depends on the strain rate dv/dr, which depends on V_{avg})
- Use the same definition as the laminar friction factor: Re=64/f

$$f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n \longrightarrow \boxed{Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n}$$

  - (Definition based on laminar Newtonian, but used for all regimes)
- Plot friction factor versus Re as for Newtonian flows, using the red definition of Re.

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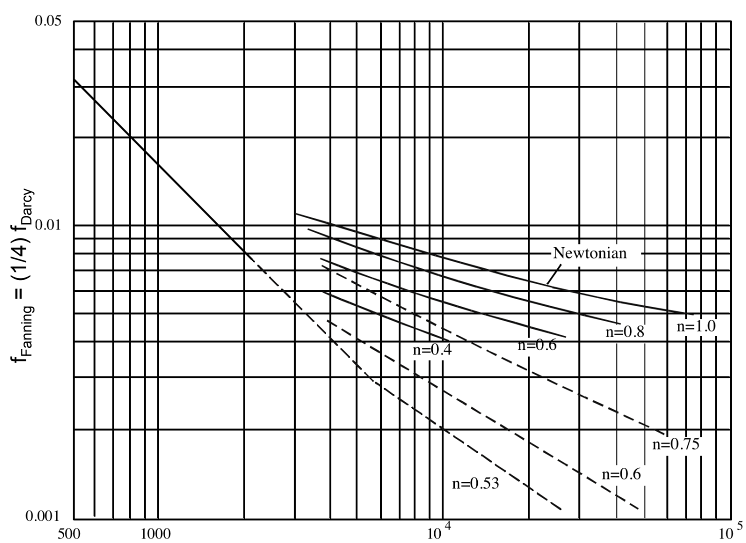
Non-Newtonian Friction Factor (Power Law)



$$Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n$$

Rheological Parameters (power law)

- **Problem:** Non-Newtonian fluid has: $\tau = K\left(-\frac{dv}{dr}\right)^n$
  - **How to find K, and n for a given fluid?**
- You need to measure something (what?)
- Try a pipe flow
  - D, Q, dP/dx
- Here's what we know:

$$\tau = K\left(-\frac{dv}{dr}\right)^n \qquad v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

$$\tau = -\frac{r}{2}\frac{dP}{dx} \qquad Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

$$\tau_w = K\left(-\frac{dv}{dr}\right)_w^n \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

$$\tau_w = -\frac{R}{2}\frac{dP}{dx}$$

- D, Q, dP/dx → V_{avg}, τ_w.
- Then relate these to K, n: $\tau_w = K\left(-\frac{dv}{dr}\right)_w^n$
  - Compute $(-dv/dr)_w$ from v(r)

Rheological Parameters (power law)

- From v(r), we get: $\left(-\frac{dv}{dr}\right)_w = \frac{2(3n+1)V_{avg}}{nD}$
- Now $\tau = K\left(-\frac{dv}{dr}\right)^n \longrightarrow \ln(\tau_w) = \ln(K) + n\ln(-dv/dr)_w$
- So a plot of $\ln(\tau_w)$ versus $\ln(-dv/dr)_w$ is linear with slope n, and intercept ln(K).
- But, note that $(-dv/dr)_w$ involves n, which is unknown → what to do?
- Just rearrange:

$$\ln(\tau_w) = \ln(K) + n\ln(2(3n+1)V_{avg}/nD)$$

$$\ln(\tau_w) = n\ln(V_{avg}) + \{\ln(K) + n\ln(2(3n+1)/nD)\}$$

- Now, a plot of $\ln(\tau_w)$ versus $\ln(V_{avg})$ is linear with slope n.
- Once n is known, K can be computed from the intercept (term in {}), or just compute it analytically from $\tau = K\left(-\frac{dv}{dr}\right)^n$ and $\left(-\frac{dv}{dr}\right)_w = \frac{2(3n+1)V_{avg}}{nD}$ which give

$$K = \frac{\tau_w}{(2(3n+1)V_{avg}/nD)^n}$$

Recap

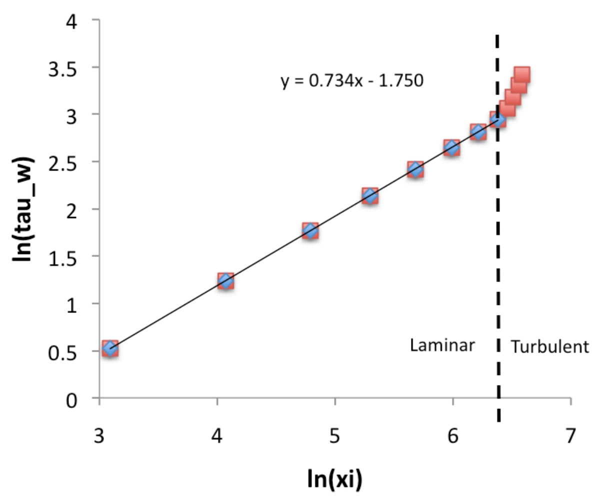
- To compute K, n for a non-Newtonian fluid
- Measure Q, D, dP/dx
- Compute V_{avg} from Q and D (area), that is, $Q=A*V_{avg}$
- Compute τ_w from $\tau_w = -\frac{R}{2}\frac{dP}{dx}$
- Plot $\ln(\tau_w)$ versus $\ln(V_{avg})$
- Fit a line to the data (the linear part of the data)
- The slope is n
- K is computed from the intercept, or from

$$K = \frac{\tau_w}{(2(3n+1)V_{avg}/nD)^n}$$

- Note, the units on K are ($kg*s^{n-2}/m$)

Example

- Given:
  - Diameter
  - Pressure Drop
  - Flow Rate
- Compute:
  - K, n
  - Re
  - Power through a given pipe is as usual, Q*ΔP



Note, here xi = $8*V_{avg}/D$, and instead of plotting $\ln(tau_w)$ versus $\ln(V_{avg})$, I'm plotting $\ln(tau_w)$ versus ln(xi). The approach is the same, but the intercept has a different formula for getting K. By the way, xi = $8*V_{avg}/D$ is $(-dv/dr)_w$ for Newtonian fluids, hence that choice here.