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From: Engineering Group
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Re: Characterizing the new heating system on the stirred tank

Introduction

Recently, the heating system on one of our stirred, short-term storage tanks was replaced, and the heat transfer properties of this new system needed to be characterized in order to develop the control system for the process. The previous system used an externally-mounted heat exchanger to maintain the temperature of the fluid in the tank at the desired value prior to delivery into the reactor. The new system uses a steam coil installed directly inside the tank. This document describes the experiments and analyses performed to determine the needed heat transfer model.

Theory

Analysis of the heat transfer characteristics in the tank begins with an energy balance. Taking the system to be the fluid in the tank, the following assumptions are made:

1. The system is stationary.
2. No mass flows into or out of the system while heat is being applied.
3. The shaft work done on the fluid by the stirrer is negligible compared to the energy transferred through heat.
4. The liquid inside the tank is incompressible.

The resulting energy balance is

$$\frac{dU_{int}}{dt} = Q \tag{1}$$

where U_{int} is the total internal energy of system, t is time, and Q is the heat flowing into the system. The heat transfer rate can be expressed by

$$Q = UA(T_s - T) \tag{2}$$

where U is the overall heat transfer coefficient between the steam and the fluid in the tank, A is the surface area available for heat transfer, T_s is the temperature of the steam, and T is the temperature of the fluid in the tank. For an incompressible fluid,

$$dU_{int} = \rho V C_p dT \tag{3}$$

where ρ is the density of the fluid, V is the volume of the fluid, and C_p is the constant pressure heat capacity of the fluid. Substituting Equations 2 and 3 into Equation 1 gives

$$\rho V C_p \frac{dT}{dt} = UA(T_s - T) \tag{4}$$

The initial condition of the system is $T = T_1$ at $t = 0$, where T_1 is the temperature of the fluid before steam is introduced into the coil. Assuming the contents of the tank are well mixed so that the temperature inside the tank is uniform and that the properties of the fluid are constant with respect to temperature, integration of Equation 4 yields

$$\ln\left[\frac{T_s - T_1}{T_s - T}\right] = \frac{UA}{\rho V C_p} t \tag{5}$$

Equation 5 is the design equation that can be used to model the heat transfer characteristics of the system. The unknown of interest in Equation 5 is U. Experiments were done to calculate U for the fluid stored in the tank. Once known, Equation 5 can be used to control the system within desired temperature ranges.

Methods

Apparatus

Figure 1 shows a schematic of the experimental apparatus. The actual storage tank and stirrer installed in the plant was used in the experiments. The tank is 66 inches tall and the inside diameter is 39 inches which gives a total capacity of 341.3 gallons. The heating coil is made of ¾-in, Schedule 40, galvanized iron pipe (O.D. 1.05 in, I.D. 0.824 in) and is 33 ft in length. The entrance of the coil is 55 inches from the bottom of the tank so that the entire coil is submerged when the tank is filled with 300 gallons of reagents—the amount currently used in the process.

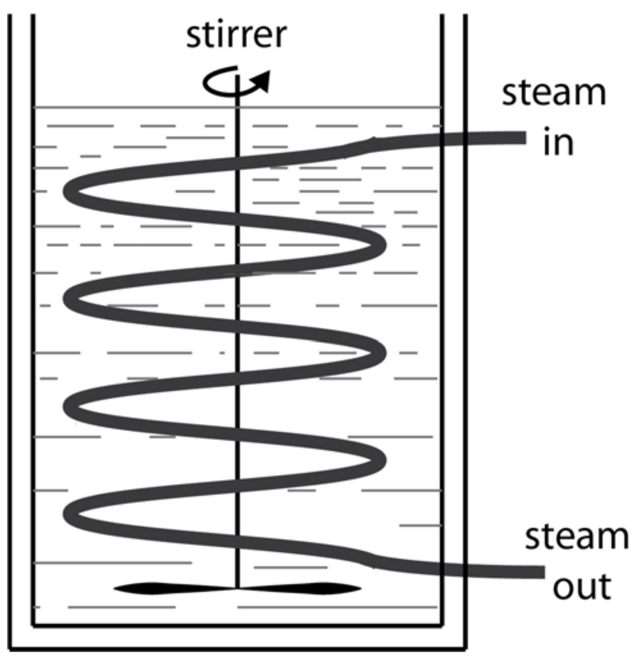

Figure 1 Storage tank with heating coil.

The height of the liquid in the tank can be read on the side of the tank. Two, type K thermocouples, obtained from the Omega company, have been installed inside of the tank to measure the temperature of the liquid. The in-house steam plant supplies the heating coil with saturated steam. The pressure of the saturated steam is regulated using a manual control valve and is measured using a pressure transducer from Data Instruments Inc. (Model # 9300101). A stirrer ensures the contents of the tank are well mixed. The entire system is controlled with Labview software which records the temperature measurements as a function of time.

Experimental Design

In order to determine the overall heat transfer coefficient for the process, experiments were performed to measure the temperature rise of the fluid as a function of time when steam is introduced into the coil. The tank was filled with 300 gallons of culinary water. After the initial temperature of the water was recorded, the steam was turned on and temperature vs. time data were collected. These data were fit to Equation 5 to obtain the value of U.

The steam pressure was set to 4 psig as this is the expected operating pressure in the plant. The atmospheric pressure in the lab during all runs was 12.5 psia. The density and heat capacity of the water were obtained from the DIPPR database and were set equal to 978.932 kg/m^3 and 4181.228 J kg^{-1} K^{-1}, respectively. A temperature of 61.5 °C, a nominal average between the initial and final temperatures of each run, was used to evaluate these two properties.

Five independent replicate experiments were done to ensure statistical significance with the initial temperature of each of the 5 replicates varying from 18.8 °C to 25.1 °C. Data were also taken at different time intervals, ranging from 30 to 90 seconds, for each experiment to reduce bias in the sampling rate. Table 1 lists the initial temperature and sampling interval of each experiment.

Table 1 Conditions for Each Experimental Run

Experiment	Initial Temperature (°C)	Sampling Interval (s)
1	21.2	30
2	18.8	40
3	23.8	50
4	25.1	65
5	19.5	90

Results and Discussion

Figure 2 shows the temperature of the water inside the tank as a function of temperature. The temperature of the water in the tank monotonically increases as a function of time as expected since the steam condensing inside the coil is at a hotter temperature than the water. Also, as expected, the rate at which the temperature of the water increases is high at initial times but continually diminishes as the temperature difference between the water and the steam inside the coil decreases. At long times, the temperature of the water approaches 103.3 °C which is the temperature of saturated steam at 16.5 psia.

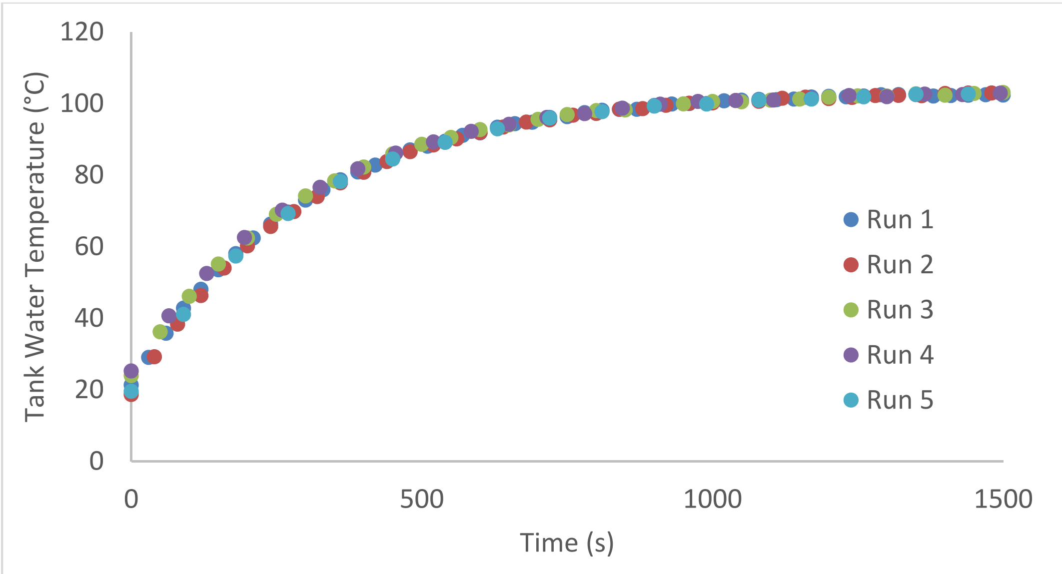

Figure 2 Tank water temperature as a function of time for each experiment.

Figure 3 shows the data from all 5 replicates transformed according to Equation 5. Specifically, the ordinate is the left hand side of Equation 5, $\ln\left[\frac{T_s-T_1}{T_s-T}\right]$, while the abscissa is $\frac{tA}{\rho V C_p}$ which is all factors on the right hand side except U. Plotted in this manner, the data follow a straight line trend where the slope of the line is U. A least-squares, linear regression was performed using all the data, and the result is also depicted on Figure 3. The best fit line gave a slope of 9237 ± 82 W m^{-2} K^{-1} where the error is the 95% confidence interval.

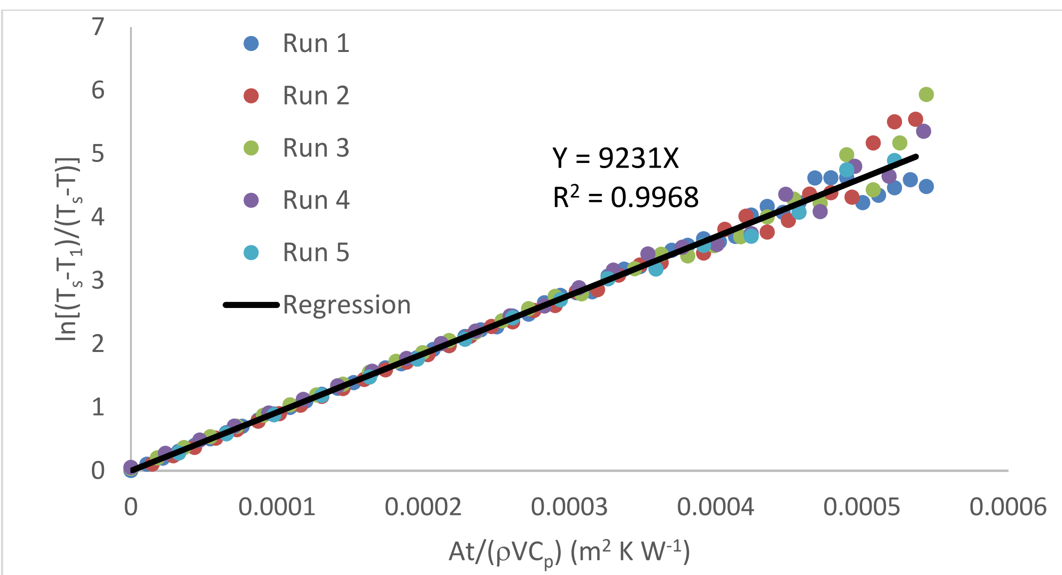

Figure 3 Fit of the experimental data to Equation 5.

The coefficient of determination of the fit (R^2) is 0.9968 which indicates the model, in general, is close to the data, but to check the adequacy of the model and ensure the prediction is not biased, Figure 4 displays the residual plot of the fit of the data. The residuals are randomly scattered about zero. The errors grow at larger values of the independent variable, but this is expected. The errors in the temperatures, as measured with a Type K thermocouples, are expected to be on the order of 2 °C. Because the dependent variable (the left-hand-side of Equation 5) contains a temperature difference that approaches zero as time passes (increases in the independent variable), but the error in the individual temperatures remains constant at 2 degrees, the residuals are expected to increase. However, because R^2 is close to 1.0 and the residuals do not indicate bias, the goodness-of-fit of the model is high.

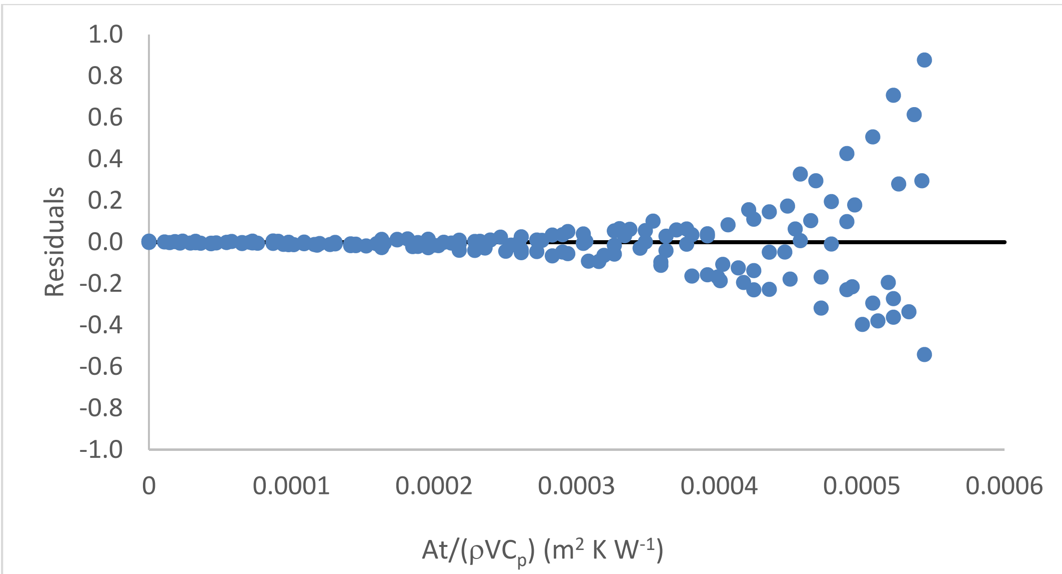

Figure 4 Residuals of the fit of the experimental data to Equation 5.

Conclusions/Recommendations

The overall heat transfer coefficient of the new heating system in the stirred storage tank is 9237 ± 82 W m^{-2} K^{-1} where the error is the 95% confidence interval. This value was obtained by regression of data from five statistically-independent experiments using the exact apparatus that will be placed in the plant. The goodness-of-fit of the experimental temperature vs. time data to the theoretical model was high as indicated by the R^2 value and the distribution of the residuals. As such, the control group can have confidence when using the model (Equation 5) when designing the control system.