MEMORANDUM

TO: Michael Beliveau

FROM: Ken Solen (representing Ken Solen and Stan Harding)

DATE: September 2, 2003

SUBJECT: Pressure Drops in our Heat Exchangers

This memo and attached report are in response to your memo of July 14 asking Stan Harding and me to evaluate a proposal to add a third heat exchanger to our antibiotic process feed stream. We have conducted measurements using a triangular channel in the laboratory (as a prototype of the channels in the exchangers) and have performed analyses relative to the proposal. The report provides details of those measurements and analyses and presents our recommendations that the proposal not be implemented unless some modification is also included.

Please feel free to contact me with questions or other discussion about this project.

Pressure Losses in Triangular Channels:
Implications for a Proposal to Add a Third Heat Exchanger
to the Antibiotic Product Feed Stream

Ken Solen

(representing Ken Solen & Stan Harding)

September 2, 2003

Abstract

Experiments and analyses were conducted to evaluate the proposal to place three compact heat exchangers in series in our antibiotic process feed stream. Pressure losses were measured for the flow of water (6 replicates at 0, 3, 6, 9, 12, and 15 L/min) through a triangular channel similar to those in the exchangers. The pressure drops increased with flow rate (with significant differences between the means for the different flow rates), but were approximately 50-100% higher than predictions from conventional calculations. This discrepancy was not explained by the observed rising temperature during the experiments or by the flow entrance length. An observed curvature in the sides of the test channel accounted for roughly half of the discrepancy, so that conventional predictions based on a corrected geometry underestimated the measured values by approximately 10-50%. Using those predictions for a flow rate of 1120 L/min (the maximum for the antibiotic feed stream) and accounting for entrance and exit losses, ΔP for three exchangers in series was estimated to be 30.3 psi. Given the entrance pressures of 27.7-31.1 psi, the required minimum exit pressure of 4.5 psi would likely not be met, and the proposed configuration was not recommended unless a pump could be introduced to increase the resulting exit pressure.

Introduction

A proposal has been made to incorporate the compact heat exchanger from the hypertension product process into the feed train of the antibiotic process, in series with the two existing exchangers (the three exchangers are identical). Stan Harding and I were asked (memo dated July 14, 2003) to determine if the cumulative pressure drop through the three exchangers (which use narrow parallel triangular channels) would exceed the limits dictated by pressure requirements in the process. Thus, experiments were to be undertaken to estimate the losses in a prototype triangular duct in the laboratory. The results of those experiments, along with the associated analyses, form the basis of this report and our recommendation concerning the abovementioned proposal.

The dimensions and important characteristics of the exchangers and feed stream provided to us are summarized in Table 1.

Table 1. Characteristics and Properties of the Relevant Equipment and Streams

Exchanger Channels:
- Cross-section: Isosceles triangles
  - equal sides: 1.31 inches,
  - base: 0.21 inches
- Surface roughness: 0.000005 inches
- Length: 7.4 feet
- Number of channels per exchanger: 192 (96 for antibiotic feed, 96 for hot oil)

Feed Stream:
- Dilute aqueous solution
- Flow rate = 1000-1120 L/min
- Temperature range: 19.8-40.3°C
- Pressure at the entrance to the heat exchanger chain: 27.7 – 31.1 psig

Required pressure after the exchangers - minimum: 4.5 psig

The goal of this investigation was to predict the overall pressure drop for the three compact heat exchangers in series and to recommend whether such a configuration should be implemented. It was clear that the evaluation should be conducted at conditions where the pressure drop would be the greatest, which would be at the maximum feed stream flow rate (1120 L/min, although measurements outside that range would help to validate the experimental results relative to predictions by conventional techniques) and the lowest temperature (19.8°C). It was also apparent that the predicted pressure drop should be compared with the smallest pressure drop allowable under the range of conditions (corresponding to the minimum inlet pressure of 27.7 psi). Finally, given the low concentrations of solutes in the antibiotic feed stream, it appeared reasonable to use water as the prototype fluid for this study.

Analytical Considerations [1]

A force balance on fluid flowing through a channel leads to the following expression for the pressure drop (ΔP) for that flow:

$$\Delta P = \tau_w L \frac{P}{A} \tag{1}$$

where
τ_w = wall shear stress
L = length of the channel
P = perimeter of the channel in contact with the fluid
A = cross-sectional area of the channel

For turbulent flow (which is the regime in which the exchangers operate), the effect of the wall shear stress is expressed through the use of the Darcy friction factor (f), which is defined as

$$f = \frac{8\tau_w}{\rho v^2} \tag{2}$$

where
ρ = density of the fluid
v = area-averaged velocity

For channels of <u>circular cross section</u>, the friction factor correlates reasonably well (±15%) with the Moody Equation:

$$\frac{1}{f^{1/2}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}_D\, f^{1/2}}\right) \tag{3}$$

where
ε = roughness of the channel wall
Re_D = Reynolds number based on diameter, which is defined as

$$\mathrm{Re}_D = \frac{Dv\rho}{\mu} \tag{4}$$

where
D = diameter of the channel
μ = viscosity of the fluid

For non–circular channels, the convention is to define the "hydraulic diameter" (D_h) as

$$D_h = 4\frac{A}{P} \tag{5}$$

so that Equations 1, 2, and 5 can be combined to give

$$\Delta P = f\frac{L}{D_h}\frac{\rho v^2}{2} \tag{6}$$

and the friction factor for Equation 6 is obtained from Equations 3 and 4, but with D_h used in place of D. However, it is recognized that this approximation becomes less accurate as the channel geometry becomes narrower (i.e., A/P decreases relative to the value for circular channels). To improve the accuracy of Equation 6, an "effective diameter" (D_{eff}) has been found to be useful to improve the estimate of f (by using it in place of D in Equations 3 and 4), where D_{eff} is the correction factor needed to cause

$$f = \frac{64}{\mathrm{Re}_{D_{eff}}} \tag{7}$$

in *laminar* flow for the geometry described. Values of D_{eff} for various geometries are available from published sources (e.g., Ref 1).

Experimental Methods

Water was circulated (5-15 L/min) from a 10-gallon reservoir through a flow circuit of 1/2"-inch ID copper pipe into which was mounted a horizontal high-alloy steel triangular duct (Figure 1). The flow rate was adjusted manually via a globe valve mounted at the outlet of the centrifugal pump (Model 14, Hydrotech, Provo, UT) at the base of the reservoir and was measured with a flowmeter (Flowmetrics, Inc., Salt Lake City, UT) located at the outlet of the triangular duct. A differential-pressure transducer (Pressodyne, Springville, UT), with taps near the entrance and exit of the duct, indicated the pressure drop for each flow rate.

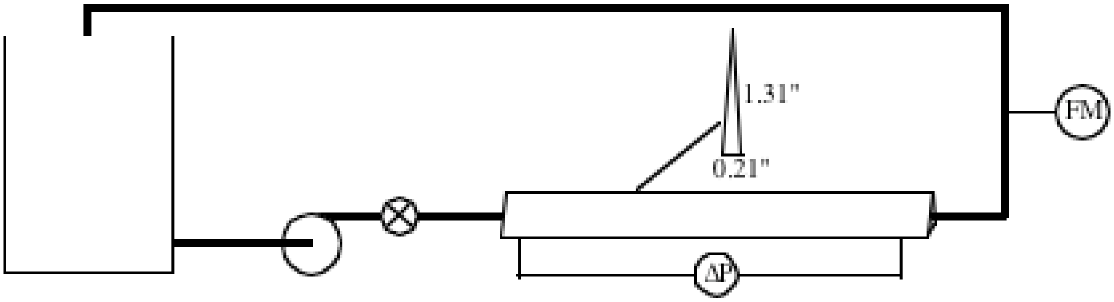


Figure 1. Schematic of the Experimental Apparatus

Measurements were made at flow rates of 0, 3, 6, 9, 12, and 15 L/min, replicated 6 times in random order. Each time the flow rate was changed to a new value for a new measurement set, the manual valve was adjusted until the flow meter indicated the target flow rate exactly (within 0.1 L/min, which was the precision of the meter indicator). At each flow rate, the experiment was conducted for 18 minutes, during which time the pressure drop, flow rate, and water temperature (in the tank) were measured every 3 minutes.

Results and Analysis

The experiments were conducted over a period of 2 days, September 10 and 12, with no mishap or apparent complications. For each change in flow rate, the pressure drops and flow rates did not change measurably after the first 30 seconds of flow, so the first measurement at 3 minutes and all subsequent measurements were considered as taken under steady-state conditions.

The temperature of the water was found to rise steadily from approximately 20°C to approximately 35°C on each of the two experiment days. This was assumed to be a result of friction heating as the water was pumped through the circuit steadily of nearly 3 hours each day. The rise in temperature is expected to cause a decrease in water viscosity, which would decrease the pressure drop through the triangular channel. Indeed, the measured pressure drops displayed a pattern in which the pressure drops for the third and sixth replicates at each flow rate (run near the end of the measurements each day) were consistently lower than those of the first and fourth replicates (run near the beginning of the experiments each day). However those differences averaged approximately 6%(in general agreement with a predicted maximum 8-9% difference using viscosities for 20 and 35°C) and were not statistically significant.

As expected, the pressure drop increased with increasing flow rate (Figure 2). In fact, mean values of ΔP at the various flow rates were significantly different ($p<0.001$) from each other (see Table 2), so random error was clearly smaller than the differences imposed by the different flow rates. The measured mean pressure drops presented in Figure 2 exceeded predictions based on the hydraulic diameter by margins ranging from 47-88% and exceeded predictions based on the effective diameter by margins ranging from 58-103%.

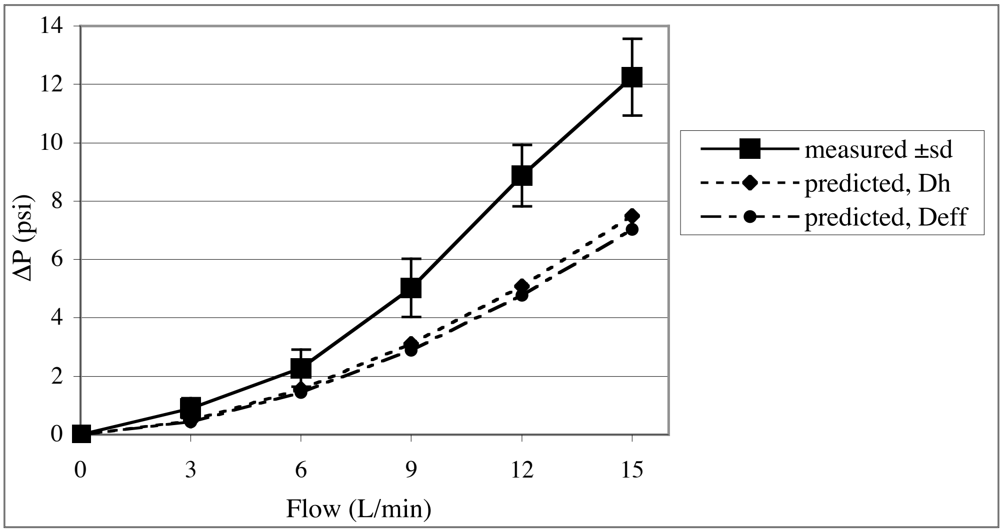


Figure 2. Pressure drops for water flowing through a triangular channel. Each measured value is the mean of 6 measurements, obtained by resetting the equipment to the target flow rate 6 times in random order and taking the mean of repeated measurements at each setting. The predicted values were obtained by conventional techniques using the hydraulic diameter (D_h) or by the additional refinement of the "effective" diameter (D_{eff}) as explained in the text.

Table 2. Statistics Comparing Mean Values of ΔP for the Various Flow Rates

Flow Rate (L/min)	0	3	6	9	12	15
Mean, ΔP	0	0.90	2.28	5.02	8.87	12.25
Standard Dev., ΔP	0	0.33	0.64	1.00	1.05	1.31
p (Student t-test)*	N/A	N/A	5.59E-16	2.4E-20	5.4E-25	1.84E-18

*Comparing the mean ΔP for that flow rate with the mean ΔP of the next lower flow rate

One possible explanation for the discrepancy between measurements and predictions is the presence of an “entrance region” where the flow is not fully developed. While the length of such a region has not been well characterized for non-circular channels, an order-of-magnitude estimate is possible using the relationship for circular channels (Equation 8)

$$\frac{L_e}{D} = 4.4\,\mathrm{Re}_D^{\;1/6} \tag{8}$$

where the channel diameter is replaced by the hydraulic diameter. That method results in an estimate of approximately 4 inches for this triangular channel. This is unlikely to contribute to the discrepancy because 1) it is less than 5% of the total length and not likely to contribute significantly to the total pressure drop and 2) it is less than the distance to the first pressure tap, implying that the flow is fully developed between the taps.

A second possible explanation for the discrepancy between the measured and predicted pressure drops is based on a warped geometry of the laboratory channel discovered upon close examination. Instead of having straight sides, the two equal sides of the isosceles triangle were discovered to be bowed in toward each other (Figure 3a). Such a distortion would provide a more narrow passageway for fluid flow in the region that already provides the greatest resistance to flow and would thereby increase the pressure drop. To estimate the magnitude of such an effect, the slope of the sides of the channel was projected from near the apex to the opposite side to create an “equivalent” triangle (Figure 3b), and the pressure drop in such a triangle was predicted using the techniques already described. The experimental measurements were found to be closer to those “corrected” predictions (10-40% relative to predictions based on D_h and 18-52% relative to predictions using the D_{eff} method) (Figure 4) than without the correction for the curved sides (Figure 2). Thus, the curvature of the sides appears to account for some of the discrepancy between the measured and predicted values. However, even with the correction, conventional predictions underestimated the pressure drop by 10-50%. Importantly, the channels in the actual heat exchangers appear from visual inspection to be “perfect” triangles (i.e. they have straight sides), and no correction would be necessary in predicting pressure losses for them.

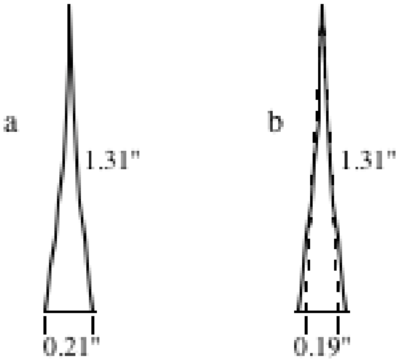


Figure 3. a) Actual shape of the triangular channel. b) Approximate “equivalent” triangle.

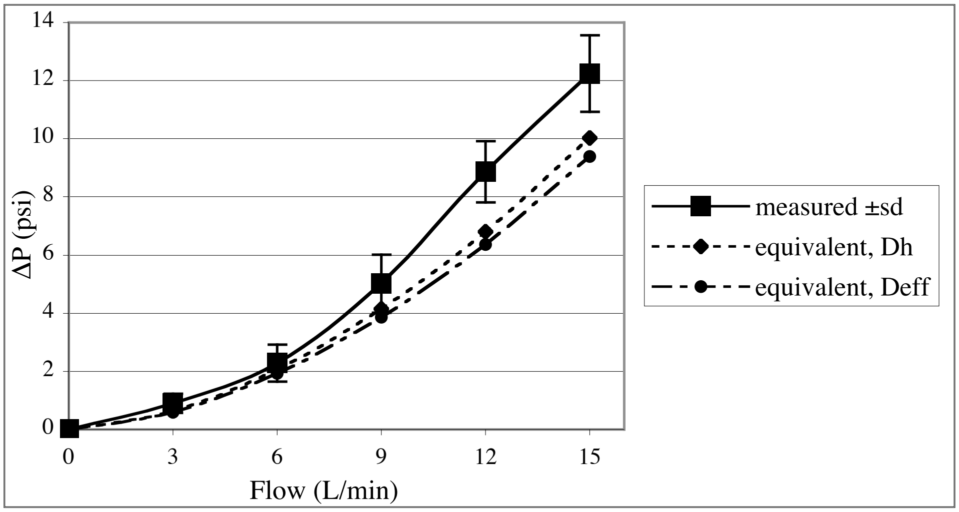


Figure 4. Measured pressure drops for water flowing through a triangular channel (see the caption for Figure 2) compared with predicted values based on a “corrected” geometry for the channel as explained in the text and illustrated in Figure 3.

The pressure drop in a series of three of our heat exchangers was estimated for a flow rate of 1120 L/min. Conventional methods previously described were used to predict the pressure drops in the triangular channels themselves, yielding values of 4.85 psi using D_h and 4.54 psi using D_{eff}. In addition, entrance and exit losses were estimated from “loss coefficients” (K) according to Equation 9

$$\Delta P_{loss} = K\frac{\rho v^2}{2} \tag{9}$$

where *K* is estimated to equal 0.5 for sharp entrances and 1.0 for large exits. The resulting estimated losses were 1.75 psi at the entrances and 3.50 psi at the exits. Thus, each exchanger would generate a maximum pressure loss of 4.85 + 1.75 + 3.50 = 10.1 psi, with a total loss of 30.3 psi for three exchangers in series.

Conclusions and Recommendations

From the experimental measurements and corresponding analysis presented here, the proposal does not appear to be consistent with process requirements. Given the pressures of 27.7-31.1 psi at the entrance of the exchangers and the predicted loss of 30.3 psi, the exit pressures would fail to meet the stated minimum requirement of 4.5 psig. This conclusion is strengthened by the observation that such predictions (even when corrected for geometry) underestimated the measured pressure drop by 10-50%.

We advise against placing the three compact heat exchangers in series in the antibiotic feed stream unless corrective action is also taken either to increase the entrance pressure or to remove the requirement of the minimum outlet pressure. Among actions to consider, we recommend the addition of a pump to the feed stream; the required additional pressure of approximately 10 psi (boosting the minimum entering pressure to 37.7 psi) is modest, and a relatively small pump would be adequate.

References

1. White FW, Fluid Mechanics, 3rd edition, NY, McGraw-Hill, 1994.

APPENDIX

MEMORANDUM

TO: Ken Solen and Stan Harding

FROM: Michael Beliveau

DATE: July 14, 2003

SUBJECT: Pressure Drops in our Heat Exchangers

As you know, the two heat exchangers used to pre-heat our antibiotic feed stream are the compact type, consisting of parallel plates with baffles between them to form small triangular channels. A third identical exchanger has just become available because we have closed down our hypertension product line, and there would be great benefit in adding this third exchanger to the antibiotic process so that the antibiotic feed stream passes through all three exchangers in series. But we're concerned that the pressure drop may be too great to provide the required exiting pressure. Because of the cost of the exchangers (recall that they are made from a titanium-nickel-chromium alloy), it would be desirable to avoid replacing these items. It also would be desirable to avoid adding a pump (it would also need to be made from corrosion-resistant alloy and both the purchase price and maintenance cost would be high). Please evaluate this situation and advise us concerning whether the re-configured system will work. For your information, the dimensions and important characteristics of the exchangers, pump, and feed stream are:

Exchanger Channels:
  Cross-section: Isosceles triangles, equal sides: 1.31 inches, base: 0.21 inches
  Surface roughness: 0.000005 inches
  Length: 7.4 feet
  Number of channels per exchanger: 192 (96 for antibiotic feed, 96 for hot oil)
Feed Stream:
  Dilute aqueous solution
  Flow rate: 1000-1120 L/min
  Temperature range: 19.8-40.3°C
  Pressure at the entrance to the heat exchanger chain: 11.7 – 14.1 psig
  Required pressure after the exchangers: minimum: 4.5 psig

The manufacturer of the heat exchangers has provided a triangular channel identical to those in the exchangers, and it has been installed in a flow circuit in our laboratory. You may find it useful for making pressure-drop and/or other measurements.

SAMPLE CALCULATION

Geometry:

Length = 7.4 ft = 226 cm
Roughness = 5.00 x 10^{-6} in = 1.27 x 10^{-5} cm

Cross-section Area:
Equal sides: 1.31 in = 3.33 cm
Base: 0.21 in = 0.533 cm
Height of triangle = side^2 – ($^1/_2$ base)^2 = (3.33 cm)^2 – (0.533 cm/2)^2 = 3.32 cm
Area = 1/2 base x height = $^1/_2$ (0.533 cm)(3.32 cm) = 0.885 cm^2

Perimeter = 2 x side + base = 2(3.33 cm) + 0.533 cm = 7.19 cm

D_h = 4 x Area/Perimeter = 4(0.885 cm^2/7.19 cm) = 0.492 cm
e/D_h = 1.27 x 10^{-5} cm /0.492 cm = 2.58 x 10^{-5}

D_{eff}:

Theta (half angle between the equal sides) = arcsin($^1/_2$ base/side) =
arcsin[0.533cm/(2x3.33cm)] = 4.60 degrees

From reference #1, Table 6.4 for isosceles triangles (see D_{eff} Correction in this appendix), f_{ReDh} is estimated to be 50.2

Thus, $D_{eff} = D_h$ x 64/ f_{ReDh} = 0.492 cm x 64/50.2 = 0.628 cm
e/D_{eff} = 1.27 x 10^{-5} cm /0.628 cm = 2.02 x 10^{-5}

Pressure Drop:

Flow = 1120 L/min = 18,670 cm^3/min
Velocity = Flow/(channels x area) = 18,670 cm^3/min/(96 x 0.885 cm^2) = 219.8 cm/s

Based on D_h

$Re_{Dh} = D_h\, v\, \rho/\mu$ = 0.492 cm x 219.8 cm/s x 0.998 g/cm^3/0.01 g/cm s = 10,800

f_{Dh} = 0.0303

$$\Delta P = f_{D_h}\frac{L}{D_h}\frac{\rho v^2}{2} = 0.0303\frac{226cm}{0.492cm}\frac{\left(0.998g/cm^3\right)\left(219.8cm\right)^2}{2}\frac{1psi}{68{,}950g/cm\,s^2} = 4.85psi$$

Based on D_{eff}

$Re_{Deff} = D_{eff}\, v\, \rho/\mu$ = 0.628 cm x 219.8 cm/s x 0.998 g/cm^3/0.01 g/cm s = 13,780

f_{Deff} = 0.0284

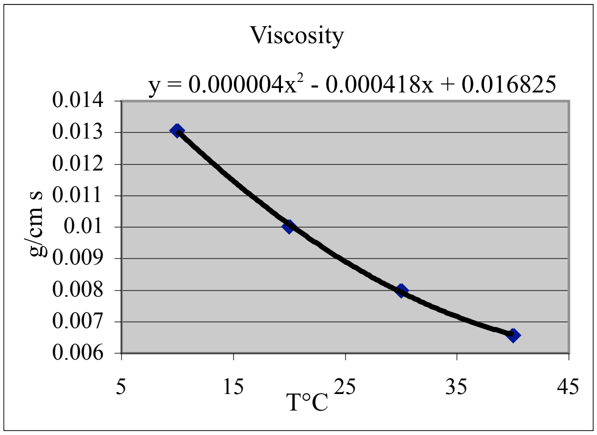
$$\Delta P = f_{D_{eff}}\frac{L}{D_h}\frac{\rho v^2}{2} = 0.0284\frac{226cm}{0.492cm}\frac{\left(0.998g/cm^3\right)\left(219.8cm\right)^2}{2}\frac{1psi}{68{,}950g/cm\,s^2} = 4.54psi$$

EFFECT OF TEMPERATURE

data from White, Table A.1

T (°C)	rho (kg/m3)	mu (Ns/m2)	rho (g/cc)	mu (g/cm s)
0	1000	0.001788	1	0.01788
10	1000	0.001307	1	0.01307
20	998	0.001003	0.998	0.01003
30	996	0.000799	0.996	0.00799
40	992	0.000657	0.992	0.00657
50	988	0.000548	0.988	0.00548
60	983	0.000467	0.983	0.00467
70	978	0.000405	0.978	0.00405
80	972	0.000355	0.972	0.00355
90	965	0.000316	0.965	0.00316
100	958	0.000283	0.958	0.00283

plotting 4 data points and adding a quadratic trend line



Input Specifications

sides (in)	1.31	sides (cm)	3.3274
base (in)	0.21	base (cm)	0.5334
length (ft)	7.4	length (cm)	225.552
roughness (in)	5.00E-06	rough (cm)	1.27E-05
density (g/cc)	0.998	density (g/cc)	0.998

h (cm)	3.31669442
Acs (sq cm)	0.8845624
Perimeter (cm	7.1882
Rh (cm)	0.12305757
Dh (cm)	0.49223027
e/Dh	2.58E-05
theta (deg)	4.59734136
fReDh	50.1594313
Deff	0.62805212
e/Deff	2.02E-05

Data

Flow	rep1-rep3	rep4-rep6
3	0.28	0.51
6	0.42	0.55
9	-0.29	0.42
12	0.21	0.64
15	0.50	0.20
ave.diff	0.34	
ave. ΔP	5.86	
%diff	5.8464311	

Heat Exchanger Calculation

flow (L/min)	1120	flow (cc/s)	18666.6667
# channels	96	# channels	96
		vel (cm/s)	219.819929

T (°C)	**20**	**T (°C)**	**35**		
visc	0.010065	visc	0.007095		
ReDh	10728.8245	ReDh	15219.9603		
1/fDh^1/2	5.74E+00	1/fDh^1/2	6.02E+00		
fDh	0.03032192	fDh	0.02763789		
deltaP (Pa)	33501.925	deltaP (Pa)	30536.4017		
deltaP (psi)	4.85901331	deltaP (psi)	4.42890319	%diff	8.85179972
ReDeff	13689.2455	ReDeff	19419.6273		
1/fDeff^1/2	5.93E+00	1/fDeff^1/2	6.21E+00		
fDeff	0.02840561	fDeff	0.02596646		
deltaP (Pa)	31384.6367	deltaP (Pa)	28689.6858		
deltaP (psi)	4.55192851	deltaP (psi)	4.16106135	%diff	8.58684749

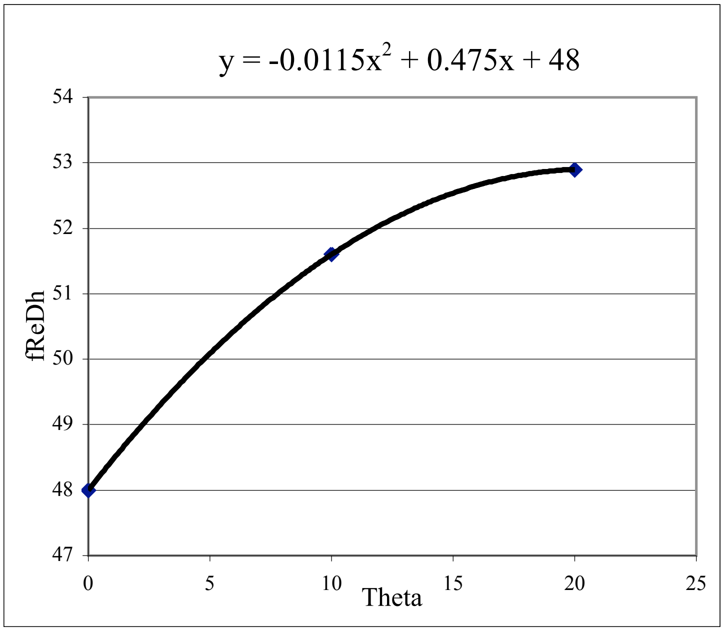
Deff CORRECTION

White (Table 6.4) provides the following for isosceles triangles:

deg	fReDh
0	48
10	51.6
20	52.9
30	53.3
40	52.9
50	52
60	51.1
70	49.5
80	48.3
90	48

where $\frac{\text{Deff}}{\text{Dh}}$ equals $\frac{64}{\text{fReDh}}$

Plotting the first 3 points and applying a quadratic "trend line" (curve fit)



EXHANGER AND CHANNEL CALCULATIONS

Input Specifications

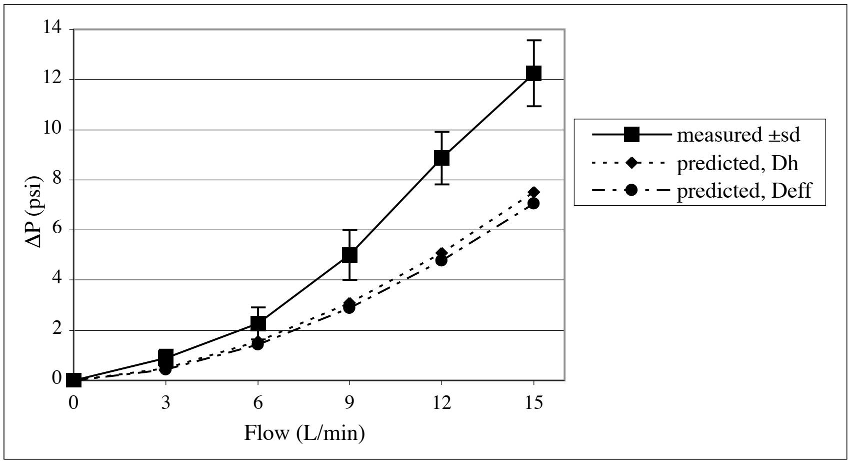
sides (in)	1.31	sides (cm)	3.3274
base (in)	0.21	base (cm)	0.5334
length (ft)	7.4	length (cm)	225.552
roughness (in)	5.00E-06	rough (cm)	1.27E-05
		h (cm)	3.316694419
		Acs (sq cm)	0.884562402
		Perimeter (cm)	7.1882
		Rh (cm)	0.123057567
		Dh (cm)	0.492230267
		e/Dh	2.58E-05
		theta (deg)	4.597341364
		fReDh	50.15943127
		Deff	0.628052119
		e/Deff	2.02E-05
viscosity (cp)	1	visc. (g/cm s)	0.01
density (g/cc)	0.998	density (g/cc)	0.998

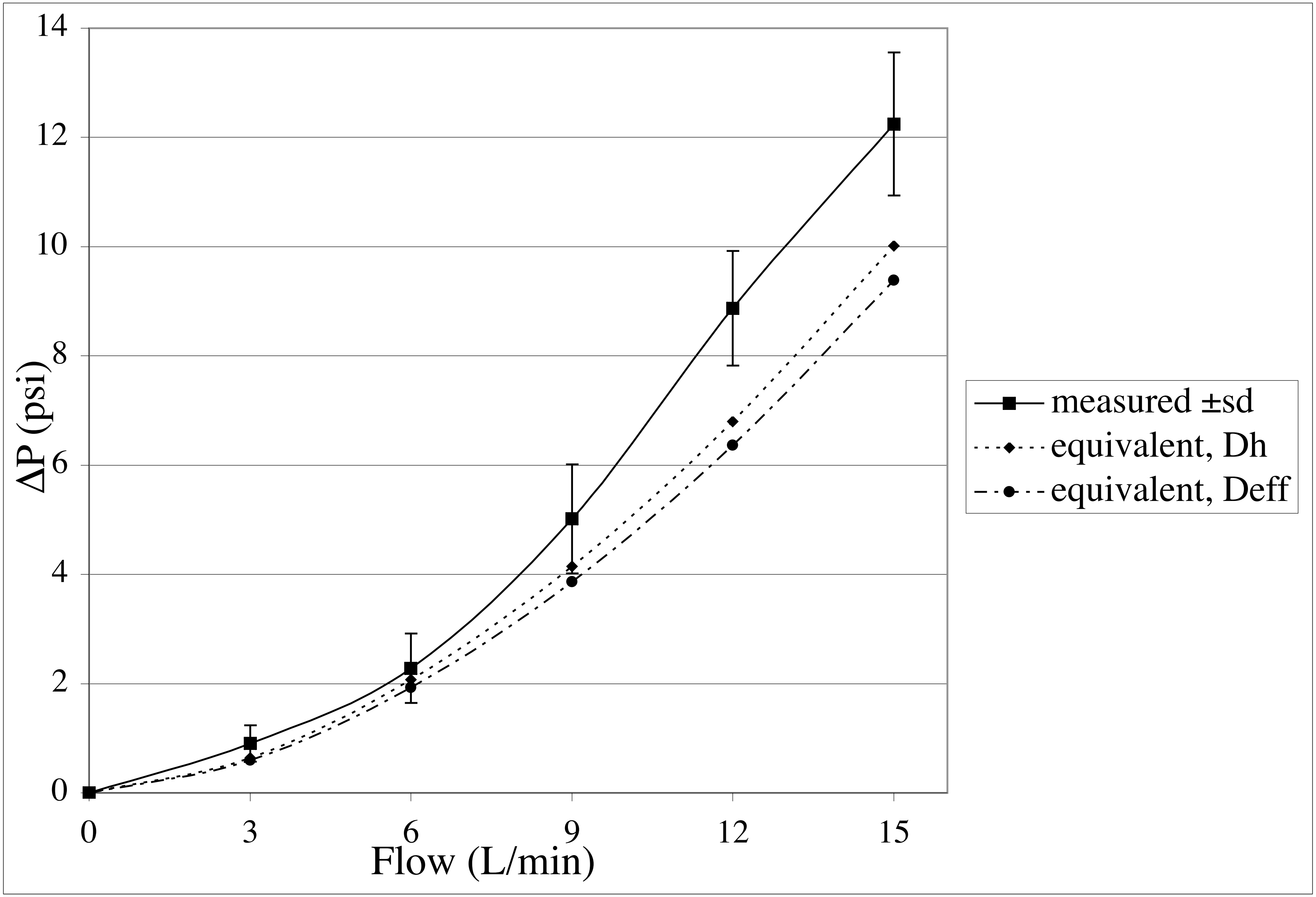
Heat Exchanger Calculation

flow (L/min)	1120	flow (cc/s)	18666.66667
# channels	96	# channels	96
		vel (cm/s)	219.8199291
		ReDh	10798.56184
		1/fDh^1/2	5.747822781
		fDh	0.030268665
		deltaP (Pa)	33443.08261
		deltaP (psi)	4.850479
		ReDeff	13778.22556
		1/fDeff^1/2	5.93837299
		fDeff	0.028357312
		deltaP (Pa)	31331.27698
		deltaP (psi)	4.544189386
		entr.loss (Pa)	12056.03991
		entr.loss (psi)	1.748569924
		exit.loss (Pa)	24112.07982
		exit.loss (psi)	3.497139848
		entrance L (cm	10.18551729
		entrance L (in)	4.010046176

Single Triangular Channel Calculation

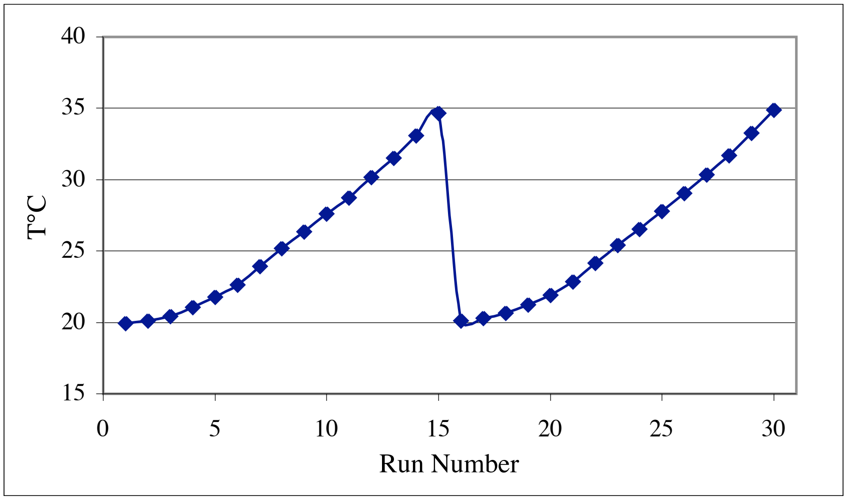
flow (L/min)	0	3	6	9	12	15
vel (cm/s)		56.52512	113.0502	169.5754	226.1005	282.6256
ReDh		2776.773	5553.546	8330.319	11107.09	13883.87
1/fDh^1/2		4.69E+00	5.23E+00	5.55E+00	5.77E+00	5.94E+00
fDh		0.045504	0.036572	0.032518	0.030039	0.028307
deltaP (Pa)		3324.409	10687.26	21380.9	35112.65	51701.03
deltaP (psi)	0	0.482162	1.550046	3.101018	5.092628	7.498554
ReDeff		3542.972	7085.945	10628.92	14171.89	17714.86
1/fDeff^1/2		4.88E+00	5.42E+00	5.74E+00	5.96E+00	6.13E+00
fDeff		0.04202	0.034046	0.030393	0.028149	0.026576
deltaP (Pa)		3069.836	9949.069	19984.05	32903.43	48538.59
deltaP (psi)	0	0.445239	1.442982	2.898423	4.77221	7.039884
Measured Data						
deltaP (psi)						
mean	0	0.90	2.28	5.02	8.87	12.25
Sd	0	0.33	0.64	1.00	1.05	1.31
% of pred,Dh		1.876416	1.469785	1.61851	1.74161	1.633167
% of pred,Deff		2.032023	1.578838	1.731641	1.858546	1.739573
t-test vs. prev.pt.		N/A	5.59E-16	2.4E-20	5.4E-25	1.84E-18




ΔP (psi)
0
2
4
6
8
10
12
14
Flow (L/min)
0
3
6
9
12
15
measured ±sd
equivalent, Dh
equivalent, Deff

PLOT TEMPERATURES

run	T avg (averaged for each run)
1	19.92
2	20.10
3	20.45
4	21.07
5	21.77
6	22.65
7	23.92
8	25.20
9	26.33
10	27.60
11	28.75
12	30.17
13	31.50
14	33.08
15	34.67
16	20.12
17	20.30
18	20.65
19	21.23
20	21.92
21	22.85
22	24.15
23	25.40
24	26.53
25	27.80
26	29.03
27	30.37
28	31.70
29	33.28
30	34.87

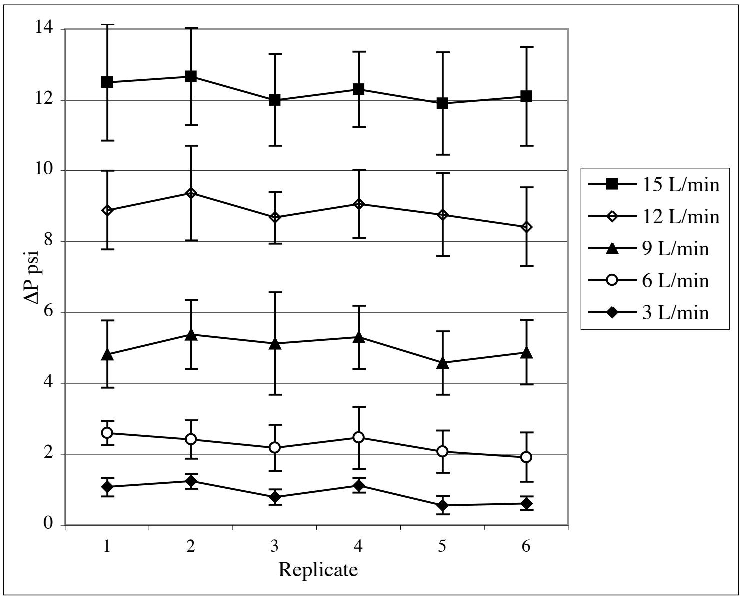


COMPILE PRESSURE DROPS

Flow (L/min)	3			6			9			12			15		
	ΔP			**ΔP**			**ΔP**			**ΔP**			**ΔP**		
	0.86	run	3	2.80	run	1	4.14	run	4	7.73	run	2	12.36	run	5
	1.33			2.76			5.77			9.98			15.30		
	0.96			1.91			4.50			8.61			11.39		
	1.35			2.79			5.85			10.06			13.35		
	0.74	mean	1.08	2.77	mean	2.60	3.47	mean	4.83	7.54	mean	8.89	10.66	mean	12.50
	1.22	sd	0.259	2.54	sd	0.349	5.27	sd	0.953	9.44	sd	1.106	11.94	sd	1.645
	1.21	run	8	3.04	run	9	4.56	run	10	7.25	run	6	13.36	run	7
	1.38			2.62			6.35			10.86			13.30		
	1.21			2.07			5.08			9.49			11.39		
	1.50			2.65			6.43			9.94			14.35		
	0.89	mean	1.24	2.63	mean	2.42	4.05	mean	5.38	8.39	mean	9.38	10.66	mean	12.67
	1.27	sd	0.206	1.50	sd	0.545	5.85	sd	0.978	10.32	sd	1.337	12.94	sd	1.377
	0.83	run	11	3.25	run	13	7.45	run	15	9.14	run	14	10.36	run	12
	1.04			2.31			5.46			8.87			12.30		
	0.67			1.76			4.19			9.50			13.39		
	0.96			2.34			5.54			8.85			13.35		
	0.45	mean	0.80	1.32	mean	2.18	3.16	mean	5.12	7.40	mean	8.68	10.66	mean	12.00
	0.83	sd	0.213	2.09	sd	0.649	4.96	sd	1.447	8.33	sd	0.735	11.94	sd	1.292
	1.12	run	18	4.07	run	20	5.43	run	17	8.96	run	19	11.73	run	16
	1.34			2.49			6.08			9.95			13.79		
	1.17			1.94			4.81			8.58			11.28		
	1.26			2.52			6.16			10.03			12.84		
	0.75	mean	1.13	1.50	mean	2.47	3.78	mean	5.30	7.48	mean	9.07	11.15	mean	12.30
	1.13	sd	0.204	2.27	sd	0.874	5.58	sd	0.894	9.41	sd	0.957	13.03	sd	1.072
	0.53	run	24	2.65	run	22	4.51	run	21	10.91	run	23	10.26	run	25
	0.45			1.91			5.40			8.80			13.20		
	0.88			1.36			4.13			7.43			11.29		
	0.77			1.94			5.48			8.88			12.25		
	0.14	mean	0.57	2.92	mean	2.08	3.10	mean	4.58	8.33	mean	8.77	10.56	mean	11.90
	0.64	sd	0.259	1.69	sd	0.591	4.90	sd	0.892	8.26	sd	1.167	13.84	sd	1.445
	0.68	run	30	1.94	run	28	5.31	run	26	9.70	run	27	13.80	run	29
	0.30			2.62			5.60			9.01			12.54		
	0.63			1.07			4.33			7.64			10.63		
	0.82			1.65			5.68			9.09			13.59		
	0.51	mean	0.62	2.83	mean	1.92	3.30	mean	4.88	6.64	mean	8.43	10.90	mean	12.11
	0.79	sd	0.194	1.40	sd	0.69	5.10	sd	0.916	8.47	sd	1.112	11.18	sd	1.397
mean	0.90			2.28			5.02			8.87			12.25		
Std. Dev.	0.334			0.636			0.997			1.049			1.309		

Trends (mean and sd of each run, grouped by flow rate)

Flow	repl–>	1	2	3	4	5	6
3	mean	1.08	1.24	0.80	1.13	0.57	0.62
	sd	0.26	0.21	0.21	0.20	0.26	0.19
6	mean	2.60	2.42	2.18	2.47	2.08	1.92
	sd	0.35	0.54	0.65	0.87	0.59	0.69
9	mean	4.83	5.38	5.12	5.30	4.58	4.88
	sd	0.95	0.98	1.45	0.89	0.89	0.92
12	mean	8.89	9.38	8.68	9.07	8.77	8.43
	sd	1.11	1.34	0.73	0.96	1.17	1.11
15	mean	12.50	12.67	12.00	12.30	11.90	12.11
	sd	1.64	1.38	1.29	1.07	1.45	1.40



RAW DATA

Sept. 10

Run	Flow		3 min	6 min	9 min	12 min	15 min	18 min
1	6	ΔP	2.80	2.76	1.91	2.79	2.77	2.54
		T	19.9	19.9	19.9	19.9	19.9	20.0
2	12	ΔP	7.73	9.98	8.61	10.06	7.54	9.44
		T	20.0	20.0	20.1	20.1	20.2	20.2
3	3	ΔP	0.86	1.33	0.96	1.35	0.74	1.22
		T	20.2	20.3	20.4	20.5	20.6	20.7
4	9	ΔP	4.14	5.77	4.50	5.85	3.47	5.27
		T	20.8	20.9	21.0	21.1	21.2	21.4
5	15	ΔP	12.36	15.30	11.39	13.35	10.66	11.94
		T	21.5	21.6	21.7	21.8	21.9	22.1
6	12	ΔP	7.25	10.86	9.49	9.94	8.39	10.32
		T	22.1	22.3	22.5	22.7	23.0	23.3
7	15	ΔP	13.36	13.30	11.39	14.35	10.66	12.94
		T	23.4	23.6	23.8	24.0	24.2	24.5
8	3	ΔP	1.21	1.38	1.21	1.50	0.89	1.27
		T	24.7	24.9	25.1	25.3	25.5	25.7
9	6	ΔP	3.04	2.62	2.07	2.65	2.63	1.50
		T	25.8	26.0	26.2	26.4	26.7	26.9
10	9	ΔP	4.56	6.35	5.08	6.43	4.05	5.85
		T	27.1	27.3	27.5	27.7	27.9	28.1
11	3	ΔP	0.83	1.04	0.67	0.96	0.45	0.83
		T	28.2	28.4	28.6	28.8	29.1	29.4
12	15	ΔP	10.36	12.30	13.39	13.35	10.66	11.94
		T	29.6	29.8	30.1	30.3	30.5	30.7
13	6	ΔP	3.25	2.31	1.76	2.34	1.32	2.09
		T	30.9	31.1	31.3	31.6	31.9	32.2
14	12	ΔP	9.14	8.87	9.50	8.85	7.40	8.33
		T	32.4	32.6	32.9	33.3	33.5	33.8
15	9	ΔP	7.45	5.46	4.19	5.54	3.16	4.96
		T	34.0	34.2	34.5	34.8	35.1	35.4

Sept. 12

Run	Flow		3 min	6 min	9 min	12 min	15 min	18 min
16	15	ΔP	11.73	13.79	11.28	12.84	11.15	13.03
		T	20.1	20.1	20.1	20.1	20.1	20.2
17	9	ΔP	5.43	6.08	4.81	6.16	3.78	5.58
		T	20.2	20.2	20.3	20.3	20.4	20.4
18	3	ΔP	1.12	1.34	1.17	1.26	0.75	1.13
		T	20.4	20.5	20.6	20.7	20.8	20.9
19	12	ΔP	8.96	9.95	8.58	10.03	7.48	9.41
		T	21.0	21.1	21.2	21.3	21.4	21.4
20	6	ΔP	4.07	2.49	1.94	2.52	1.50	2.27
		T	21.5	21.7	21.8	22.0	22.2	22.3
21	9	ΔP	4.51	5.40	4.13	5.48	3.10	4.90
		T	22.3	22.5	22.7	22.9	23.2	23.5
22	6	ΔP	2.65	1.91	1.36	1.94	2.92	1.69
		T	23.6	23.8	24.0	24.2	24.5	24.8
23	12	ΔP	10.91	8.80	7.43	8.88	8.33	8.26
		T	24.9	25.1	25.3	25.5	25.7	25.9
24	3	ΔP	0.53	0.45	0.88	0.77	0.14	0.64
		T	26.0	26.2	26.4	26.6	26.9	27.1
25	15	ΔP	10.26	13.20	11.29	12.25	10.56	13.84
		T	27.3	27.5	27.7	27.9	28.1	28.3
26	9	ΔP	5.31	5.60	4.33	5.68	3.30	5.10
		T	28.5	28.7	28.9	29.1	29.4	29.6
27	12	ΔP	9.70	9.01	7.64	9.09	6.64	8.47
		T	29.8	30.0	30.3	30.5	30.7	30.9
28	6	ΔP	1.94	2.62	1.07	1.65	2.83	1.40
		T	31.1	31.3	31.5	31.8	32.1	32.4
29	15	ΔP	13.80	12.54	10.63	13.59	10.90	11.18
		T	32.6	32.8	33.1	33.5	33.7	34.0
30	3	ΔP	0.68	0.30	0.63	0.82	0.51	0.79
		T	34.2	34.4	34.7	35.0	35.3	35.6